

# Solving the Quantum Nonlocality Enigma by Weyl's Conformal Geometrodynamics.

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## Abstract

Since the 1935 proposal by Einstein Podolsky and Rosen the riddle of nonlocality, today demonstrated by innumerable experiments, has been a cause of concern and confusion within the debate over the foundations of quantum mechanics. The present paper tackles the problem by a non relativistic approach based on the Weyl's conformal differential geometry applied to the Hamilton-Jacobi solution of the dynamical problem of two entangled spin 1/2 particles. It is found that the nonlocality rests on the entanglement of the spin internal variables, playing the role of "hidden variables". At the end, the violation of the Bell inequalities is demonstrated without recourse to the common nonlocality paradigm. A discussion over the role of the "*internal space*" of any entangled dynamical system involves deep conceptual issues, such the *indeterminism* of quantum mechanics and explores the in principle limitations to any exact dynamical theory when truly "hidden variables" are present. Because of the underlying geometrical foundations linking necessarily gravitation and quantum mechanics, the theory presented in this work may be considered to belong to the unifying "quantum gravity" scenario.

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## I. INTRODUCTION

Since the 1935 publication of the famous paper by Einstein – Podolsky – Rosen (EPR), the awkward coexistence within the quantum lexicon of the contradictory terms “*locality*” and “*nonlocality*” as primary attributes to quantum mechanics (QM) has been a cause of concern and confusion within the debate over the foundations of this central branch of modern Science [1]. In particular, even today this paradigmatic conundrum keeps eliciting animated philosophical quarrels. For instance, an extended literature consisting of articles and books is produced today by eminent quantum field theorists endorsing the “local” side of the dilemma by a “*principle of locality*” based on the premise that quantum observables measured in mutually spacelike separated regions commute with one other [2–4]. On the other hand, the confirmation by today innumerable experiments, following the first one by Alain Aspect and coworkers, of the violation of the Bell inequalities emphasizes the dramatic content of the dispute [5–7]. By referring to the implications of Relativity with the nonlocal EPR correlations the philosopher Tim Maudlin writes: “*One way or another, God has played us a nasty trick. The voice of Nature has always been faint, but in this case it speaks of riddles and mumbles as well...*” [8]. Indeed, as it has been well known for three decades, the experimental violation of the Bell inequalities implies the existence of quite “mysterious” nonlocal correlations linking the outcomes of the measurements carried out over two spatially distant particles. Moreover, in spite of these correlations any superluminal transfer of useful information is found to be forbidden according to a “no-signalling theorem”. Recently this was even tested experimentally [9].

Aimed at a clarification of the problem, the present article tackles the well known EPR scheme and explains by an exact analysis the violation of the Bell’s inequalities through a non relativistic approach, for simplicity. Two equal spin-1/2 particles  $A$  and  $B$ , e.g. two neutrons, propagate in opposite directions along the spatial  $y$ -axis ( $\vec{y}$ ) of the Laboratory with a velocity  $v \ll c$  towards two spatially separate measurement devices, dubbed Alice and Bob, who measure the spin of  $A$  and  $B$ , respectively. Each apparatus, measuring the particle  $A$  (or  $B$ ), consists of a standard Stern-Gerlach (SGA) device followed by a couple of particle detectors (D) that, being rigidly connected to SGA, can be oriented with it by a rotation in the  $\vec{x}$ - $\vec{z}$  plane at the corresponding angles  $\theta_A$  (or  $\theta_B$ ) taken respect to  $\vec{z}$ .

Accordingly,  $\vec{\theta}_A$  and  $\vec{\theta}_B$  denote the orientation axes of  $\text{SGA}_A$  and  $\text{SGA}_B$  [7].

## II. THE NON RELATIVISTIC QUANTUM TOP AND WEYL'S CURVATURE

Keeping the validity of the “principle of locality” which eventually refers to the final measurement on the spins, it appears clear that the main problem implied by quantum non-locality resides with the very “ontological” nature of the wavefunction  $\psi(A, B)$  that links the two particles since their common spatial origin. This problem involves the “completeness” status of  $\psi$ , its deep implications with the “Schrödinger Cat” Paradox and possibly the solutions offered by several exotic theories, e.g. the one based on the “ $\psi$  spontaneous collapse” [10], the Everett’s “Many Worlds” [11] or the Albert-Loewer “Many Minds” interpretations [12]. Indeed, the quantum correlation affecting any particle measurement, e.g. the outcome obtained by Bob once the corresponding one has been obtained by Alice (or viceversa), is a factual event, implying a temporary (or permanent) mechanical (or electrical) change of the very structure of a physical measuring device. Of course, a structural change cannot be achieved if the “link” provided by  $\psi(A, B)$  were a purely informational entity, as assumed by a fairly large number of scientists. In facts, any measurement outcome or any probability being a “number”, cannot by itself determine a structural change on any physical apparatus. As already stressed in previous papers, this, or similar problems strongly suggest that the wavefunction is not a mere mathematical entity but consists of a physical “field” and, more precisely, as we shall see, a “gauge field” acting within a quantum theory based on Weyl’s conformal differential geometry [13]. Accordingly, the present theory of the EPR process is Weyl-conformally invariant [14, 15].

Let’s consider, within a “classical” framework, a single particle, say  $A$ , consisting of a spinning spherical top of mass  $m$  and inertial moment  $I_c = ma^2$ ,  $a$  being the top’s gyration radius<sup>1</sup>. The top configuration space, with dimensions  $n = 6$ , is given by the direct product of the “*external space*” of the Laboratory  $\{x^i\}$ , spanned by the top’s center of mass coordinates  $x^i = \{x, y, z\}$ , and of the “*internal space*”  $\{\zeta^\alpha\}$  spanned by the Euler’s angles:  $\zeta^\alpha = \{\alpha, \beta, \gamma\}$  i.e. the top’s “*internal variables*” fixing its orien-

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<sup>1</sup> The radius  $a$  is not determined in the present theory; however, when the theory is extended to the relativistic framework, the radius  $a$  is fixed by the theory itself to be of the order of the particle Compton wavelength [15].

tation in space  $\{x^i\}$ . The coordinates in the configuration space  $\{q^\mu\}$  are then  $q^\mu = \{x, y, z, \alpha, \beta, \gamma\}$ , ( $\mu = 1, \dots, 6$ ). The kinetic energy of the top is:  $K = \frac{1}{2}(mv^2 + I_c\omega^2) = \frac{1}{2} mg_{\mu\nu}\dot{q}^\mu\dot{q}^\nu$  where the spatial components of the velocity vectors are:  $v^i = \{\dot{x}, \dot{y}, \dot{z}\}$  and  $\omega^i \doteq \lambda_\alpha^i \dot{\zeta}^\alpha = \left\{ -\dot{\beta} \sin \alpha + \dot{\gamma} \cos \alpha \sin \beta, \dot{\beta} \cos \alpha + \dot{\gamma} \sin \alpha \sin \beta, \dot{\alpha} + \dot{\gamma} \cos \beta \right\}$  [16]. The tensor  $g_{\mu\nu}$ , with determinant:  $g = a^6 \sin^2 \beta$ , has a diagonal 2-block form, i.e.: (a) a  $3 \times 3$  block:  $g_{ij} = \delta_{ij}$  the diagonal Euclidean metric of the flat  $\{x^i\}$  space, (b) a  $3 \times 3$  block  $g_{\alpha\beta} = a^2 \gamma_{\alpha\beta}$  with  $\gamma_{\alpha\beta}$  the symmetric, nondiagonal Euler metric tensor of the internal space  $\{\zeta^\alpha\}$ . The internal metric  $g_{\alpha\beta}$  exhibits a constant Riemann curvature:  $R = 3/(2a^2)$ . The quantities  $\lambda_\alpha^i$  introduced in the given expression of  $\omega^i$  can be considered as the parameters of a set of three congruences in the internal space allowing the Euler metric  $\gamma_{\alpha\beta}$  to be written in the dyadic form  $\gamma_{\alpha\beta} = \lambda_\alpha^i \lambda_\beta^i$  together with its inverse  $\gamma^{\alpha\beta} = \mu_i^\alpha \mu_i^\beta$ , where  $\mu_i^\alpha \lambda_\beta^i = \delta_\beta^\alpha$  and  $\mu_i^\alpha$  are the momenta of the congruences. The spin-1/2 operators components  $\hat{s}_i = (\hbar/2)\hat{\sigma}_i$  on the spatial axes  $\{x^i\}$  ( $\hat{\sigma}_i$  are Pauli's operators) are introduced as derivatives along line arcs:  $\hat{s}_i \doteq -i\hbar\mu_i^\alpha \partial_\alpha$ . The standard spin commutation relations hold:  $[\hat{\sigma}_i, \hat{\sigma}_j] = 2i\epsilon_{ijk}\hat{\sigma}_k$ , as it may be checked by a direct calculation [17].

The top configuration space is now endowed with the Weyl's connection implied by the parallel transport of vectors of differential geometry  $\Gamma_{\mu\nu}^\sigma = -\left\{ \begin{smallmatrix} \sigma \\ \mu\nu \end{smallmatrix} \right\} + \delta_\mu^\sigma \phi_\nu + \delta_\nu^\sigma \phi_\mu + g_{\mu\nu} \phi^\sigma$  where  $g_{\mu\nu}$  is the metric tensor,  $\left\{ \begin{smallmatrix} \sigma \\ \mu\nu \end{smallmatrix} \right\}$  the Christoffel symbols and  $\phi_\mu = \partial_\mu \phi$  is a Weyl's vector assumed to be integrable, i.e. a gradient of a *Weyl's scalar potential*  $\phi$  [13, 18]. The quantity  $\phi_\mu$  has been identified as a cosmological "World vector potential" by Peter G. Bergmann, the renowned Einstein's scholar and collaborator [19]. The dynamics of the top is realized considering the Lagrangian  $L$  formed by adding to the kinetic energy  $K$  a potential proportional to the Weyl curvature:  $L = K - \frac{\xi\hbar^2}{m}R_W$ , where  $R_W = R + (n-1)[(2/\sqrt{g})\partial_\mu(\sqrt{g}g^{\mu\nu}\phi_\nu) - (n-2)g^{\mu\nu}\phi_\mu\phi_\nu]$ . In the last equations,  $R$  is the Riemann curvature and, furthermore,  $\xi$  is a numerical coupling parameter given by  $\xi = [(n-2)/8(n-1)] = 1/10$ . The constant  $\hbar^2$  implies that in the present theory the potential  $R_W$  is the source of all quantum features of the system. The Hamilton-Jacobi equation (HJE) of the top's dynamics is:  $-\partial_t S = \frac{1}{2m} g^{\mu\nu} \partial_\mu S \partial_\nu S + \frac{\xi\hbar^2}{m} R_W$ , where the "Action"  $S$  is the Hamilton's principal function. The trajectories of the top in the configuration space are given by the generalized velocities  $v^\mu = \dot{q}^\mu = \frac{1}{m} g^{\mu\nu} p_\nu$ , where  $p_\mu = \partial_\mu S$  are the generalized momenta. In order to determine the potential  $\phi$ , the HJE must be solved consistently with the *continuity equation*  $\partial_t \rho + \frac{1}{\sqrt{g}} \partial_\mu (\sqrt{g} \rho v^\mu) = 0$ , where the "density" is related to the Weyl potential

$\phi$  by  $\rho = A \exp[-(n-2)\phi]$ ,  $A$  being a normalization constant. The last equation can be used to express the Weyl curvature  $R_W$  in terms of  $\rho$  obtaining  $R_W = R + [(n-1)/(n-2)][g^{\mu\nu} \partial_\mu \rho \partial_\nu \rho / \rho^2 - (2/\rho \sqrt{g}) \partial_\mu (\sqrt{g} g^{\mu\nu} \partial_\nu \rho)]$ . Previous articles reported a complete quantum solution of the present problem in a fully relativistic framework, leading to a new, "ab initio" derivation of the Dirac's equations [15]. The key result of this approach is that, unlike the metric which is fixed, Weyl's potential  $\phi$ , as well as Weyl's curvature  $R_W$ , are determined by the top's motion. This motion, in turn, is affected by  $R_W$ : a typical self-reacting geometrodynamical process well known in the context of General Relativity [20]. In other words, owing to the self-effect of  $R_W$  the single, apparently isolated spinning particle can never be considered "free": as we shall see soon, this unavoidable self-interaction is the basic geometrical origin of the quantum nonlocality. For space limitations we report here the final results of the solution adapted to the present nonrelativistic approach for "free" particles [15]. By means of the *ansatz*:  $\psi(q^\mu, t) = \sqrt{\rho(q^\mu, t)} \exp[iS(q^\mu, t)/\hbar]$ , here interpreted as a "scalar wave function" satisfying Born's rule:  $\rho = |\psi|^2$ , the coupled problem implied by the continuity and by the HJE equations is fully linearized - this is the startling key result of the overall theory - and cast in the form of the standard first-quantization theory based on the Schrödinger-De-Rahm wave equation:

$$i\hbar \partial_t \psi = -\frac{\hbar^2}{2m\sqrt{g}} \partial_\mu (\sqrt{g} g^{\mu\nu} \partial_\nu \psi) + \frac{\xi \hbar^2}{m} R \psi, \quad (1)$$

where  $R = 3/(2a^2)$  is the constant Riemann scalar curvature calculated from the metric  $g_{\mu\nu}$ . Because of the linearity, this wave equation is adopted, as usual, to describe the dynamical vector evolution of the quantum state of the system in the standard Hilbert space [21]. The ensuing theory reproduces the standard quantum theory in all formal details, in spite of the new dynamical interpretation of  $\psi(q^\mu)$ . As a quite remarkable feature, the Weyl's vector  $\phi_\mu(q^\mu)$  and potential  $\phi(q^\mu)$  formally disappear from the wave equation as they are kept hidden in the very definitions of  $\rho(q^\mu)$  and  $S(q^\mu)$ : this may explain why this or a similar theory based on conformal Weyl's symmetry was never previously formulated. In facts, the starting Lagrangian  $L$  and all relevant equations occurring in the theory are invariant under the Weyl's gauge transformations  $g_{\mu\nu} \rightarrow \lambda g_{\mu\nu}$ ,  $\phi_\mu \rightarrow \phi_\mu + (1/2\lambda) \partial_\mu \lambda$  provided the values of the "Weyl's weight",  $w$  of the action  $S$  and of the mass  $m$  are:  $w(S) = 0$  and  $w(m) = -1$ , respectively. As a consequence the velocity fields  $v^\mu$ , the particle trajectories, the scalar density  $m\rho\sqrt{g}$  and current  $j^\mu = m\rho\sqrt{g}v^\mu$  have weight:  $w = 0$  i.e. they are all

Weyl gauge invariant. Also the eigenvalues  $E$  of the Schrödinger-De-Rahm equations given by:  $i\hbar\partial_t\psi = E\psi$  are gauge invariant. The particle mass is not gauge invariant, but the mass ratio is gauge invariant, which is all we need to have well defined mass values<sup>2</sup>.

The general solution of the wave equation (1) for the spin  $\frac{1}{2}$  particle can be cast in the form

$$\psi(q^\mu, t) = e^{-i\Omega t} [D_\uparrow(\alpha, \beta, \gamma)\psi_1(x, y, z, t) + D_\downarrow(\alpha, \beta, \gamma)\psi_2(x, y, z, t)], \quad (2)$$

where  $\Omega = 21\hbar/(40ma^2)$ ,  $D_\uparrow(\alpha, \beta, \gamma) = e^{\frac{1}{2}i(\alpha+\gamma)} \cos \frac{\beta}{2} = (\hat{D}^{-1}(\alpha, \beta, \gamma))_{11}$ ,  $D_\downarrow(\alpha, \beta, \gamma) = e^{-\frac{1}{2}i(\alpha-\gamma)} \sin \frac{\beta}{2} = (\hat{D}^{-1}(\alpha, \beta, \gamma))_{12}$  are the entries of the inverse of the Wigner's SU(2) matrix  $\hat{D}(\alpha, \beta, \gamma)$  representing the 3D-space rotation  $\hat{R}_3(\alpha, \beta, \gamma)$ , and  $\psi_1(x, y, z, t)$  and  $\psi_2(x, y, z, t)$  are solutions of the time-dependent Schrödinger equation of the *free* particle with mass  $m$  [16]. We notice once again that the particle appears to be free in the wave equation, because the geometric self-action of the particle on itself due to the Weyl curvature is completely hidden in the structure of the wavefunction  $\psi$ . Under space rotation  $\{x^i\} \rightarrow \hat{R}_3\{x^i\}$ , the wavefunction  $\psi(q^\mu, t)$  changes as a scalar field, which implies that the fields  $\psi_1(x, y, z, t)$  and  $\psi_2(x, y, z, t)$  change as the two components of the Pauli spinor  $\tilde{\psi}(x, y, z, t) = \begin{pmatrix} \psi_1(x, y, z, t) \\ \psi_2(x, y, z, t) \end{pmatrix}$ . The spinor components  $\psi_1(x, y, z, t)$  and  $\psi_2(x, y, z, t)$  correspond to the usual quantum states of the particle with spin aligned in the positive and negative direction of the Laboratory  $z$ -axis, respectively. In this way, spin 1/2 fields obeying the free particle time-dependent Schrödinger equation appear naturally in the theory, although the overall conformal invariance symmetry is lost when a purely spinorial approach is used. In a more familiar notation, Eq. (2) is written as  $\psi(q^\mu) \equiv \langle x, y, z, \alpha, \beta, \gamma | \psi_{spin}(t) \rangle$ , where  $|\psi_{spin}(t)\rangle = |\psi_1(t), \uparrow_z\rangle + |\psi_2(t), \downarrow_z\rangle$ . The normalization is taken as  $\langle \psi_{spin}(t) | \psi_{spin}(t) \rangle = \int dx dy dz (|\psi_1(x, y, z, t)|^2 + |\psi_2(x, y, z, t)|^2) = 1$ , where the integration over the Euler angles was carried out with the usual measure:  $d\mu(\alpha, \beta, \gamma) = \frac{1}{4\pi^2} \sin \beta d\alpha d\beta d\gamma$ .

### III. QUANTUM ENTANGLEMENT AND WEYL'S CURVATURE

The extension to two or more spinning particles is straightforward. In the case of two spins  $A$  and  $B$ , the configuration space has  $n = 12$  dimensions and is the direct product

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<sup>2</sup> The mass enters as a parameter in the present non relativistic approach, which is carried out in the gauge where  $m = \text{const.}$ ; however, when the theory is extended to the relativistic framework, the particle mass is no longer an external parameter, but it becomes a consequence of the theory itself [15].

of the configurations spaces of the two particles, spanned by the generalized coordinates  $q^\mu = (x_A, y_A, z_A, \alpha_A, \beta_A, \gamma_A; x_B, y_B, z_B, \alpha_B, \beta_B, \gamma_B)$  ( $\mu = 1, \dots, 12$ ). The kinetic part of the total lagrangian  $L_{AB}$  is the sum of the kinetic parts of the Lagrangians of the two particles and the metric  $g_{\mu\nu}$  induced in the configuration space has still a block diagonal form where the space and angle degrees of freedom are separated. The Riemann curvature of the metric  $g_{\mu\nu}$  is the sum of the Riemann curvature of the two single particle subspaces: its value,  $R = 3/a^2$  doesn't contribute to any inter-particle correlations (we assume equal mass and equal inertia moments for the two particles). However, the Weyl curvature  $R_W(q^\mu)$  for the two particles does not split, in general, into separate contributions. Indeed, besides the geometrical self-action of each particle, an interaction between the two particles is established. This interaction disappears only in a very particular case, i.e. in absence of quantum entanglement, as we shall see shortly. We realize, therefore, that the very true origin of all effects related to quantum entanglement, including the Bell's inequalities violation, resides in the not eliminable interaction due to the presence of the Weyl curvature in the Lagrangian. Formally, the key point consists of the logarithmic dependence of the Weyl potential  $\phi$  on the quantity  $\rho = |\psi|^2$  according to:  $\phi(q^\mu) = -(n-2)^{-1} \ln(\rho)$ ; ( $n = 12$ ). In the absence of entanglement, we have  $\psi = \psi_A \psi_B$  and  $\rho = \rho_A \rho_B$ , so that  $\phi = \phi_A + \phi_B$ , where  $\phi_A$  (or  $\phi_B$ ) depends on the coordinates of particle  $A$  (or  $B$ ) only. Consequently, the Weyl curvature splits into  $R_W = R_W(A) + R_W(B)$ , the solution of the HJE splits into  $S = S_A + S_B$ , the velocity field splits into  $v^\mu = v_A^\mu + v_B^\mu$  and the overall continuity equation splits into two separated continuity equations. On the other hand, when the wavefunction  $\psi$  cannot be separated, the same nonseparability occurs for  $R_W$ ,  $S$ , the velocity field, and the continuity equation. We emphasize once again that the underlying interaction due to the Weyl curvature is manifest in the HJE of the present theory, but it is completely hidden in the quantum wave equation, which merely reduces to the sum of the separate wave equations of the two particles. It follows that we cannot ascertain the presence of entanglement just looking at the wave equation itself: we must look instead at the form of its solutions. At the level of the equations of the theory, entanglement is unveiled as a true physical phenomenon due to a non trivial space Weyl's curvature only within the context of the present approach. The key point is that different solutions of the wave equations lead to different Weyl's curvatures and, hence, to a different interaction among the orientational degrees of freedom of the two particles. As we shall see by a paradigmatic example in the next Section, the very origin of

quantum nonlocality relies on this unavoidable orientational geometric interaction.

#### IV. THE EPR SCALAR WAVEFUNCTION FOR TWO SPIN 1/2 PARTICLES

Let us consider the EPR rotational invariant, "singlet" quantum state of the particles  $A$  and  $B$  in motion along the  $y$ -axis:

$$|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}[|\uparrow_z^A, \downarrow_z^B\rangle - |\downarrow_z^A, \uparrow_z^B\rangle]. \quad (3)$$

The scalar wavefunction corresponding to this state is easily found to be

$$\psi_{AB}(q^\mu, t) \equiv \frac{1}{\sqrt{2}} e^{i(\frac{\gamma_A + \gamma_B}{2} - 2\Omega t)} \left[ e^{-i\frac{\Delta\alpha}{2}} \cos \frac{\beta_A}{2} \sin \frac{\beta_B}{2} - e^{i\frac{\Delta\alpha}{2}} \cos \frac{\beta_B}{2} \sin \frac{\beta_A}{2} \right] \times \quad (4)$$

$$\times \psi_A(x_A, y_A, z_A, t) \psi_B(x_B, y_B, z_B, t), \quad (5)$$

where  $\Delta\alpha = (\alpha_B - \alpha_A)$ . We don't consider Pauli's exclusion principle here, because the *external-space* wavefunctions  $\psi_A(x^i)$  and  $\psi_B(x^i)$  are supposed to be associated to two well separated wavepackets at positions  $y_A$  and  $y_B$  on the  $y$ -axis, respectively. The action and the density associated to  $\psi_{AB}(q^\mu, t)$  are

$$S(q^\mu, t) = \hbar[-2\Omega + \frac{\gamma_A + \gamma_B}{2} + \arctan\left(\csc \frac{\beta_A - \beta_B}{2} \sin \frac{\beta_A + \beta_B}{2} \tan \frac{\alpha_B - \alpha_A}{2}\right) + \arg(\psi_A(x_A^i)) + \arg(\psi_B(x_B^i))] \quad (6)$$

and

$$\rho(q^\mu, t) = \frac{1}{4} |\psi_A(x_A^i)|^2 |\psi_B(x_B^i)|^2 [1 - \cos \beta_A \cos \beta_B - \cos(\Delta\alpha) \sin \beta_A \sin \beta_B]. \quad (7)$$

The differential equations of motion  $\dot{q}^\mu = (1/m)g^{\mu\nu}\partial_\nu S$  derived from (6) splits into three decoupled sets: one involving the center of mass coordinates of particle  $A$  only, one involving the center of mass coordinates of particle  $B$  only, and a third set involving the Euler's angles of both  $A$  and  $B$ . As said, the last set of equations cannot be decoupled because of quantum entanglement. The presence of entanglement is also unveiled by the expression of the Weyl curvature  $R_W$  derived from Eq. (7):

$$R_W^{(AB)} = \frac{48}{5a^2} + \frac{22}{5a^2(1 - \cos \beta_A \cos \beta_B - \cos \Delta\alpha \sin \beta_A \sin \beta_B)} + R_W^{(A)}(x_A, y_A, z_A, t) + R_W^{(B)}(x_B, y_B, z_B, t) \quad (8)$$

where  $R_W^{(A)}$  and  $R_W^{(B)}$  are the Weyl spacetime curvatures associated with the fields  $\psi_A$  and  $\psi_B$ , respectively. We see again that the total Weyl curvature is equally splitted into three



terms: the curvatures  $R_W^{(A)}$  and  $R_W^{(B)}$ , depending on the space-time "external" coordinates and yielding the self-action of each particle on itself, and the coupling term which depends on the Euler angles only. Unlike the spacetime terms, this last term cannot be splitted into the sum of two independent potentials acting on each particle and it is the responsible of all phenomena related to quantum entanglement. In this way, the very nature of entanglement is explained as originating from the residual coupling of the orientational degrees of freedom of the two spins due to the presence of the Weyl's curvature  $R_W^{(AB)}$ . This one is the origin of a inter-particle coupling consisting of a real orientational force that one particle exerts on the other. As said, this force originates from  $R_W^{(AB)}$ , which in turn originates from Weyl's potential  $\phi_{AB}$ , which ultimately arises from the system's wavefunction:  $\psi_{AB}$ . This last one then loses its meaning of a purely mathematical entity in favor of the more pregnant concept of a physical field. To summarize, in the presence of entanglement, the *internal coordinates*  $\{\zeta_{A/B}^\alpha\}$ , viz. the Euler angles of the tops  $A$  and  $B$ , cannot be disentangled *irrespective* of the mutual spatial distance separating the two travelling particles. Even if these ones are space-like separated by a large distance  $d$ , an inter-particle coupling independent of  $d$  arises that cannot be eliminated and is responsible for the *nonlocal EPR correlations*. Furthermore, we conjecture from the present nonrelativistic standpoint that the space-time superluminality of the nonlocal correlations comes from the geometrical independency, i.e. disconnectedness, of the internal and external manifolds:  $\{x^i\}$  and  $\{\zeta^\alpha\}$ . This is the key result of the present Article. Note that the dynamical Euler's angle coupling disappears in the *absence of entanglement*, i.e. in the case of a "product-state". For instance, for the state:  $|\uparrow_z^A, \downarrow_z^B\rangle$  the term depending on the Euler angles in the Weyl's curvature is:  $R_W = -\frac{11}{5a^2} \left( \frac{1}{1+\cos\beta_1} + \frac{1}{1-\cos\beta_2} \right)$ , i.e. the contributions of  $A$  and  $B$  are mutually independent and separable.

In summary, when the entanglement is present, in order to save the completeness of the theory the dynamical phase-space of any quantum system must consist of the tensor product of the "external space-time" and of the "internal space" viz. the one spanned by the *internal variables*, which may be interpreted as (non measurable) dynamical "hidden variables". This appears to be at variance with the methods of standard quantum theory where the internal variables are commonly integrated away, e.g. within the process of definition of the "spin", which is itself a measurable quantity, for instance by a SGA device. In this respect, we may conjecture that the (often necessary) overlooking of the *internal space* leads to

several consequences of deep conceptual and philosophical relevance. For instance, we believe that some relevant manifestations of "*quantum indeterminism*" as well as the "*quantum nonlocality*" precisely arise from an over-simplified treatment of the dynamical problem, i.e. from the neglect or the lack of knowledge of the internal variables of the system. All this may draw us into even more profound speculations. Since in our World - or in our Universe since the Big-Bang - all objects, bodies or elementary particles are entangled because of the continuous, enormous wealth of mutual interactions, even the "external" space-time geometry of Special or General Relativity cannot be assumed, in principle, as a reliable background of a complete dynamical theory. In the limit, no dynamics, no mechanics, no physics would be possible but for systems brought, ironically, into an "unphysical" isolation condition, e.g. in the case of few isolated ions confined in an electromagnetic Paul trap [22]. Of course it has been known for centuries that the solution of any physical problem always implies the adoption of an idealized "filtering" of approximate dynamical conditions: e.g. a successful study of the motion of the moon around the earth must neglect the effect of the far galaxies. However, the paradigmatic entanglement case accounted for in the present Article resists to any approximation. Indeed, as we have seen, the mere neglect in the theory of the manifold made of the "internal variables" - classified as "hidden" because assumed to be inaccessible to measurement - leads irreducibly to an unconceivable result, i.e. to the riddle of "quantum nonlocality" revealed by the quite "mysterious" violation of the Bell's inequalities.

## V. THE MEANING OF QUANTUM MEASUREMENT

To better understand why the internal variables are not directly accessible to experiments, we need a closer view of how experiments are carried out in the quantum world. In essence, any experimental apparatus designed to measure some physical property of a quantum particle is made of two parts: a "filtering" device which addresses the particle to the appropriate detector channel according the possible values of the quantity to be measured (e.g. a spin component) and one (or more) detectors able to register the arrival of the particle. To fix the ideas, we consider here the particular case of the measure of a spin  $1/2$  particle by a Stern-Gerlach (SGA) apparatus. The spin component along the SGA axis can have two values, so we need two detectors  $D_u$  and  $D_d$  coupled to the "up"

and "down" output channels of the orientable SGA. Each detector measures the flux  $\Phi$  of particles entering its acceptance area  $A$ . Let's assume single particle detection. Then this flux is given by  $\Phi = \int_A j^\mu n_\mu dA = \int_\Sigma \rho \sqrt{g} g^{\mu\nu} \partial_\nu S n_\mu d\Sigma$  extended to the hypersurface  $\Sigma$  in the particle configuration space with normal unit vector  $n_\mu = n_\mu = \{\mathbf{n}, 0, 0, 0\}$  where  $\mathbf{n}$  is the usual 3D-normal to the detector area  $A$ . Let us assume that the scalar wavefunction of the particle at the detector location has its spacetime and angular parts factorized, i.e.  $\psi = \psi_1(x, y, z, t) \psi_2(\alpha, \beta, \gamma)$ . Then  $\rho = \rho_1(x, y, z, t) \rho_2(\alpha, \beta, \gamma)$ ,  $S = S_1(x, y, z, t) + S_2(\alpha, \beta, \gamma)$  and  $\Phi = \int_A \mathbf{j} \cdot \mathbf{n} dA \int \rho_2(\alpha, \beta, \gamma) d\mu(\alpha, \beta, \gamma)$ , where  $\mathbf{j} = \rho_1(x, y, z, t) \nabla S_1$ . The particle flux  $\Phi$  is the only quantity directly accessible to the detector and depends only on the spacetime part  $\psi_1(x, y, z, t)$  of the wavefunction. The Euler's angles are integrated away for the simple reason that the detector is located in the physical space-time. It is worth noting that the current density  $j^\mu$  and, hence, the flux  $\Phi$  is Weyl-gauge invariant as it must be for any quantity having a measurable value.

Let us consider now the role played by the filtering apparatus. Unlike the detector, whose role is just to count particles, the filtering stage of the experimental setup must be tailored on the quantity to be measured. In the case of the SGA the filtering device is the spatial orientation of the inhomogeneous magnetic field crossed by the particle's beam. In an ideal filtering apparatus no particle is lost, so its action on the particle's wavefunction is unitary. The role of the filter is to correlate the spacetime path of the particle with the quantity to be measured (the spin component, in our case) so to extract from the incident beam all particles with a given value of the quantity (spin "up", for example) by addressing them to the appropriate detector. The filter acts on the particle motion in space-time only. But, as said before, there is a feedback between the particle motion and the geometric curvature of the space, so that the insertion of the filter changes not only the particle path in spacetime, but also the overall geometry of the particle configuration space, because it modifies its Weyl's curvature  $R_W$  through  $\phi_\mu$ , the, according to Bergmann, environmental *World vector potential* [19]. This mechanism is at the core of General Relativity: the change in the motion or the addition of a massive body changes the geometry of the whole surrounding space. In our present approach, both particle motion and space geometry are encoded in the scalar wavefunction, which indeed changes under the action of the "unitary", i.e. lossless, transformation introduced by the SGA filter. Solving the full dynamical and geometric problem inside the SGA is a difficult problem, but the asymptotic behavior of

the scalar wavefunction far from the SGA may be easily found. In this "far-field scattering approximation", a uniformly polarized particle beam is transformed by a SGA rotated at angle  $\theta$  with respect to the  $\vec{z}$ -axis as follows,

$$\begin{aligned}
& [aD_{\uparrow}(\alpha, \beta, \gamma) + bD_{\downarrow}(\alpha, \beta, \gamma)]\psi(x, y, z, t) \xrightarrow{SGA} \\
& \left(a \cos \frac{\theta}{2} + b \sin \frac{\theta}{2}\right) \left(D_{\uparrow}(\alpha, \beta, \gamma) \cos \frac{\theta}{2} + D_{\downarrow}(\alpha, \beta, \gamma) \sin \frac{\theta}{2}\right) \psi(x_u, y_u, z_u, t) + \\
& + \left(a \sin \frac{\theta}{2} - b \cos \frac{\theta}{2}\right) \left(D_{\uparrow}(\alpha, \beta, \gamma) \sin \frac{\theta}{2} - D_{\downarrow}(\alpha, \beta, \gamma) \cos \frac{\theta}{2}\right) \psi(x_d, y_d, z_d, t) \quad (9)
\end{aligned}$$

where  $a, b$  are arbitrary complex constants with  $|a|^2 + |b|^2 = 1$ , and labels "u" and "d" refer to the positions of the detectors located to the up and down exit channels of the  $\theta$ -oriented SGA. The experimental apparatus is arranged so that the wave packets  $\psi(x_u, y_u, z_u, t)$  and  $\psi(x_d, y_d, z_d, t)$  have negligible superposition so that each detector sees a wavefunction with space and angular parts factorized. Thus, for example, the particle flux detected in the "up" channel of the SGA is given by  $\Phi_u P_u(\theta)$ , where  $\Phi_u$  is the particle flux on the detector and  $P_u(\theta) = |a \cos \frac{\theta}{2} + b \sin \frac{\theta}{2}|^2$  is usually interpreted as the probability that the particle in the input wavepacket is found with its spin along the "up" direction of the SGA.

What the filter does is to correlate the particle space-time trajectory with the quantity to be measured. In the standard quantum mechanical language, we may say that the filter introduces a controlled entanglement among the quantity to be measured and the particle spacetime path (in the SGA case, the spacetime degrees of freedom become entangled with the orientational ones). However, the filter is configured so that the wavepackets arriving on each detector ( $D_u$  and  $D_d$ , in our case) are not superimposed, and the (approximate) wavefunction seen by each detector is of the product form as considered above. The last requirement ensures that the detected particle flux  $\Phi$  provides a correct measure (in the quantum sense) of the measured quantity<sup>3</sup>.

## VI. THE EPR STATE AND BELL INEQUALITIES

Let's now turn our attention to the joint spin measurements of the EPR entangled particles  $A$  and  $B$  described by Eq. (4). After leaving the source, particles  $A$  and  $B$  travel

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<sup>3</sup> It is precisely the lack of this condition which prevents to use the SGA to measure the spin of electrons. A way to overcome this fundamental limitation was proposed very recently [23].

towards two Stern Gerlach setups,  $\text{SGA}_A$  and  $\text{SGA}_B$ , respectively, located at Alice's and Bob's stations on two distant sites along  $\vec{y}$ . As said before, each SGA acts *locally*, by a *unitary* transformation, on the particle spatial, i.e. *external*, degrees of freedom by correlating its exit direction of motion with the direction of its spin respect to the SGA axis, rotated around  $\vec{y}$  at angle  $\theta$ , taken respect to  $\vec{z}$ . Since we are dealing with  $\frac{1}{2}$ -spins, there are only two exit directions, either "up" or "down" available to each particle which will be then finally registered by a corresponding detector. Let's refer to the Alice's and Bob's detectors as  $D_{Au}$ ,  $D_{Ad}$ ,  $D_{Bu}$ ,  $D_{Bd}$  and let  $\theta_A$  and  $\theta_B$  the angles of  $\text{SGA}_A$  and  $\text{SGA}_B$ , respectively. Labels "u" or "d" refer to the particle's exit directions from each SGA's. As said above, the presence of the two SGA changes not only the trajectories of the two particles, but also the Weyl curvature of their configuration space. These changes are both encoded in the change of the wavefunction  $\psi_{AB}$  in Eq. (4). Near the source that wavefunction remains approximately unchanged, but far beyond the spatial positions of the two SGA's the paths of the particles acquire different direction according to their spin so that near the locations of the detectors the input wavefunction is transformed according to

$$\begin{aligned} \psi_{AB} \xrightarrow{\text{SGAs}} & A_{u,u} \psi_A(\mathbf{r}_{Au}, t) \psi_B(\mathbf{r}_{Bu}, t) + A_{u,d} \psi_A(\mathbf{r}_{Au}, t) \psi_B(\mathbf{r}_{Bd}, t) + \\ & + A_{d,u} \psi_A(\mathbf{r}_{Ad}, t) \psi_B(\mathbf{r}_{Bu}, t) + A_{d,d} \psi_A(\mathbf{r}_{Ad}, t) \psi_B(\mathbf{r}_{Bd}, t) \end{aligned} \quad (10)$$

where  $\mathbf{r}_{Au}$ ,  $\mathbf{r}_{Ad}$ ,  $\mathbf{r}_{Bu}$ ,  $\mathbf{r}_{Bd}$  are the positions of the detectors and  $A_{u,u}$ ,  $A_{u,d}$ ,  $A_{d,u}$ ,  $A_{d,d}$  are coefficients depending on the two particle Euler's angles and on the angles  $\theta_A$  and  $\theta_B$  of  $\text{SGA}_A$  and  $\text{SGA}_B$ , respectively. The coefficients  $A$  can be easily calculated by applying Eq.

(9):

$$A_{u,u} = \chi(t) \left( D_{\uparrow}(\alpha_1, \beta_1, \gamma_1) \cos \frac{\theta_A}{2} + D_{\downarrow}(\alpha_1, \beta_1, \gamma_1) \sin \frac{\theta_A}{2} \right) \times \\ \times \left( D_{\uparrow}(\alpha_2, \beta_2, \gamma_2) \cos \frac{\theta_B}{2} + D_{\downarrow}(\alpha_2, \beta_2, \gamma_2) \sin \frac{\theta_B}{2} \right) \sin \Delta\vartheta \quad (11a)$$

$$A_{u,d} = \chi(t) \left( D_{\uparrow}(\alpha_1, \beta_1, \gamma_1) \cos \frac{\theta_A}{2} + D_{\downarrow}(\alpha_1, \beta_1, \gamma_1) \sin \frac{\theta_A}{2} \right) \times \\ \times \left( -D_{\uparrow}(\alpha_2, \beta_2, \gamma_2) \sin \frac{\theta_B}{2} + D_{\downarrow}(\alpha_2, \beta_2, \gamma_2) \cos \frac{\theta_B}{2} \right) \cos \Delta\vartheta \quad (11b)$$

$$A_{d,u} = \chi(t) \left( -D_{\uparrow}(\alpha_1, \beta_1, \gamma_1) \sin \frac{\theta_A}{2} + D_{\downarrow}(\alpha_1, \beta_1, \gamma_1) \cos \frac{\theta_A}{2} \right) \times \\ \times \left( D_{\uparrow}(\alpha_2, \beta_2, \gamma_2) \cos \frac{\theta_B}{2} + D_{\downarrow}(\alpha_2, \beta_2, \gamma_2) \sin \frac{\theta_B}{2} \right) \cos \Delta\vartheta \quad (11c)$$

$$A_{d,d} = \chi(t) \left( -D_{\uparrow}(\alpha_1, \beta_1, \gamma_1) \sin \frac{\theta_A}{2} + D_{\downarrow}(\alpha_1, \beta_1, \gamma_1) \cos \frac{\theta_A}{2} \right) \times \\ \times \left( -D_{\uparrow}(\alpha_2, \beta_2, \gamma_2) \sin \frac{\theta_B}{2} + D_{\downarrow}(\alpha_2, \beta_2, \gamma_2) \cos \frac{\theta_B}{2} \right) \sin \Delta\vartheta \quad (11d)$$

where:  $\chi(t) \equiv \frac{1}{\sqrt{2}}e^{-2i\Omega t}$  and:  $(\Delta\vartheta) \equiv (\theta_B - \theta_A)/2$ . The coincidence rate are given by the joint particle fluxes intercepted by the detectors, viz.  $\Phi_{i,j}(\theta_A, \theta_B) = \iint |A_{ij}(\alpha_1, \beta_1, \gamma_1, \alpha_2, \beta_2, \gamma_2; \theta_A, \theta_B)|^2 d\mu(\alpha_1, \beta_1, \gamma_1) d\mu(\alpha_2, \beta_2, \gamma_2) \int \mathbf{j}_i \cdot \mathbf{n}_i dA_i \int \mathbf{j}_j \cdot \mathbf{n}_j dA_j$ , where  $i, j = u, d$  and:  $\mathbf{j}_i = |\psi_A(\mathbf{r}_i, t)|^2 \nabla S_A(\mathbf{r}_i, t)$ ,  $\mathbf{j}_j = |\psi_B(\mathbf{r}_j, t)|^2 \nabla S_B(\mathbf{r}_j, t)$  are the particle current densities at the detectors. A simple calculation shows that if all particles falling into the detectors are counted, the coincidence fluxes are given by  $\Phi_{u,u}(\theta_A, \theta_B) = \Phi_{d,d}(\theta_A, \theta_B) = \frac{1}{2} \sin^2(\Delta\vartheta)$  and  $\Phi_{u,d}(\theta_A, \theta_B) = \Phi_{d,u}(\theta_A, \theta_B) = \frac{1}{2} \cos^2(\Delta\vartheta)$ . The coincidence fluxes  $\Phi_{ij}$  are Weyl-invariant and can be experimentally measured. Moreover, they are equal to the joint probabilities  $P_{i,j}(\theta_A, \theta_B)$  associated with the EPR state (4), in full agreement with the standard quantum theory and lead straightforwardly to the violation of the Bell's inequalities within all appropriate experiments consisting of statistical measurements over several choices of the angular quantity  $(\Delta\vartheta)$ , as shown by many modern texts [6–8, 24]. For instance, Redhead considers the inequality:  $F(\Delta\vartheta) \equiv |1 + 2 \cos(2\Delta\vartheta) - \cos(4\Delta\vartheta)| \leq 2$  which is violated for all values of  $(\Delta\vartheta)$  between  $0^\circ$  and  $45^\circ$ . In summary, in order to attain the correct result the present theory promoted the "hidden variables" to the status of "internal variables" of the particle's relevant property: the "spin". We believe that these variables, i.e. the Euler's angles, should be considered as a necessary dynamical aspects of that fundamental quantum property. Note that the standard "hidden variables" no-go theorem is not violated by our

theory, because in standard quantum theory the not trivial curved configuration space and the feedback between space curvature and particle motion are absent [21].

## VII. CONCLUSIONS

We have demonstrated that the quantum nonlocality enigma, epitomized by the violation of the Bell's inequalities, may be understood on the basis of a Weyl's conformal geometrodynamics. This result was reached through a theory that bears several appealing properties and may lead to far reaching consequences in modern physics. We summarize them as follows:

- 1) The linear structure of the standard first quantization theory is fully preserved, in any formal detail.
- 2) The quantum wavefunction acquires the precise meaning of a physical quantum "*Weyl's gauge field*" acting in a curved configurational space.
- 3) A proper theoretical analysis of any quantum *entanglement* condition must involve the entire configurational space of the system including the usual space-time of General Relativity as well as the "internal coordinates" of the system. If entanglement is present and if the internal coordinates are really "hidden", i.e. if they are absent in the theory - as they are in standard quantum theory - severe limitations may arise on the actual interpretation of any dynamical problem. There physics may even be an impossible task, in principle, and paradoxes may spring out. Indeed, in addition to "*quantum nonlocality*", many counterintuitive concepts of quantum mechanics, such as those related to several aspects of "*quantum indeterminism*" and of "*quantum counterfactuality*" may precisely arise from these theoretical limitations. Which are indeed limitations to the human knowledge and understanding.
- 4) The "sinister", "disconcerting" and "discomforting" aspects of entanglement were expressed right after the publication of the EPR paper by a surprised and highly concerned Erwin Schrödinger. Who also added: "I would not call that one but rather *the* characteristic trait of quantum mechanics, the one that enforces the departure from the classical lines of thought" [25].
- 5) By solving an utterly important enigma the present paper clarifies *the* - according to Schrödinger - characteristic trait of quantum mechanics. The adopted theory is based on a

necessary significant aspect of the interplay between geometry and matter motion on which also rests the modern theory of gravitation, i.e. General Relativity. Consequently, our work may be considered to belong to a unifying theoretical scenario linking necessarily gravitation and quantum mechanics. This is indeed the long sought, paradigmatic "quantum gravity" scenario.

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